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LETTER TO THE EDITOR

Localization of two-level systems driven by AC–DC fields

Hao Wang and Xian-Geng Zhao

Institute of Theoretical Physics, Academia Sinica, PO Box 2735, Beijing 100080, People's Republic of China

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Abstract. The quantum behaviour of electrons in a two-level system under the influence of an AC–DC field is investigated. The closed-form solution of quasienergy is obtained, from which the exact level crossing is found to appear at a fan structure in the section $(E,\ E_0)$ through the parameter space $(E,\ E_0,\ \omega)$, where E is the magnitude of the AC field, E_0 is the magnitude of the DC field, and ω is the frequency of the AC field. It is demonstrated that the emergence of the exact crossing of quasienergies is coincident with the onset of coherent destruction of tunnelling.

Recently there has been considerable interest in the time evolution in driven two-level systems [1–4]. The reason is twofold. On the one hand, the system is simple but has strong background and potential application in different fields [5]. On the other hand, the theoretical findings for the system, if any, can be fulfilled through the observation of quantum coherent behaviour of electrons (or excitons) in double-well heterostructures, because such a double-well heterostructure can be fairly well reproduced by a two-level approximation [6].

As is well known, for a double-well heterostructure in the absence of driving forces, the electron can visit either well due to the quantum tunnelling. However, if the double-well structure is biased, i.e, driven by a DC field, the electron in an eigenstate will be naturally localized in one of the wells because of the suppression of the coherent tunnelling between two wells [7]. The situation in which the double-well system is exposed to a pure AC field, e.g., a laser field, is more interesting. There it was found numerically that if the ratio of the reduced field strength to the field frequency was a root of the ordinary Bessel function of order zero, an initially localized electron in one of the wells would remain localized in that well permanently, as if the electron were 'frozen' by the field [8]. This phenomenon of the coherent destruction of tunnelling is believed to be coincident with the onset of the quasienergy crossing.

A natural next step is to look at what happens if both the DC field and the AC field are switched on the double-well heterostructure simultaneously. We are especially concerned with whether the dynamic localization can still be emergent, and if so, which conditions determine the choice of field parameters. These questions form our motivation for studying this problem in the present work. The Hamiltonian we considered here can be approximately written as

$$H(t) = \Delta\sigma_z + \mu[E_0 + E\cos\omega t]\sigma_x \tag{1}$$

where σ_z and σ_x (as well as σ_y used below) are the Pauli matrices, $\Delta\sigma_z$ is the unperturbed Hamiltonian of the system with eigenvalues $(E_\alpha = -\Delta,\ E_\beta = \Delta)$ and eigenvectors $(|\alpha\rangle,\ |\beta\rangle)$, μ is the transition dipole between two levels, E_0 is a constant field for breaking the symmetry of the double well, and E and ω are, respectively, the amplitude and the frequency of the driving laser field. This model was studied recently for the special case of weak interlevel coupling and high-frequency driving field [9]. Of late, the general case has also been discussed [10].

It has been shown in [10] that the study of the Hamiltonian (1) can be carried out analytically with the help of SU(2) symmetry. As a result, the closed-form solution of the evolution operator can be obtained generally. In the present work, by applying this method, we study the quasienergy spectrum for the system, focusing on the effect of dynamic localization for an electron moving in such a two-level system driven by the AC–DC field. We find that when one ratio of the reduced DC field strength to the photon energy becomes an integer, i.e., $2\mu E_0/\omega = n$ ($n = 0, 1, 2, \ldots$), the exact quasienergy crossing will occur if the other ratio of the AC field strength to the photon energy, $2\mu E/\omega$, is a root of the ordinary Bessel function of nth order simultaneously. This means that the coherent tunnelling of electrons between the two biased wells may completely turn to that of localized motion by the choice of field parameters. Such findings are confirmed by our numerical calculations for the evolution probability. Now, we report our findings as follows.

The Hamiltonian (1) is periodic in time with period $T = 2\pi/\omega$, $H(t+T) = H(t)$. Hence, Floquet's theorem asserts that the wavefunctions of $H(t)$ can be written as $\psi(t) = \exp(-\mathrm{i}\epsilon t)\phi(t)$ with the time-periodic Floquet functions $\phi(t+T) = \phi(t)$; here ϵ is the quasienergy, and $\phi(t)$ satisfy the Schrödinger equation (we take $\hbar = 1$ throughout this work)

$$\left[H(t) - \mathrm{i}\frac{\partial}{\partial t}\right]\phi(t) = \epsilon\phi(t). \tag{2}$$

Following the method presented in [10], and after applying some techniques, we obtain the closed-form solution for the quasienergy as

$$\epsilon_\pm = \pm\frac{\omega}{2\pi}\Theta(T)\ \mathrm{mod}(\omega) \tag{3}$$

with

$$\Theta(T) = \cos^{-1}\left\{\cos\left(2\pi\frac{\mu E_0}{\omega}\right)\sum_{m=0}^{\infty} R_x^{(2m)}(T,0) + \sin\left(2\pi\frac{\mu E_0}{\omega}\right)\sum_{m=0}^{\infty} R_y^{(2m)}(T,0)\right\} \tag{4}$$

$$R_x^{(2m)}(t,t_0) = (-1)^m\left(\prod_{l=1}^{2m}\int_{t_0}^{t}\mathrm{d}t_l\right)\theta(t_1-t_2)\ldots\theta(t_{2m-1}-t_{2m})X(t_1t_2\ldots t_{2m}) \tag{5}$$

$$R_y^{(2m)}(t,t_0) = (-1)^m\left(\prod_{l=1}^{2m}\int_{t_0}^{t}\mathrm{d}t_l\right)\theta(t_1-t_2)\ldots\theta(t_{2m-1}-t_{2m})Y(t_1t_2\ldots t_{2m}) \tag{6}$$

$$R_x^{(0)}(t,t_0) = 1 \qquad R_y^{(0)}(t,t_0) = 0 \tag{7}$$

where $\theta(t) = 1$ for $t > 0$ and 0 otherwise; $X(t_1t_2\ldots t_m)$ and $Y(t_1t_2\ldots t_m)$ satisfy the following recurrence formulas

$$X(t_1t_2\ldots t_m) = X(t_1t_2\ldots t_{m-1})X(t_m) + Y(t_1t_2\ldots t_{m-1})Y(t_m) \qquad (m \geqslant 2) \tag{8}$$

$$Y(t_1t_2\ldots t_m) = X(t_1t_2\ldots t_{m-1})Y(t_m) - Y(t_1t_2\ldots t_{m-1})X(t_m) \qquad (m \geqslant 2). \tag{9}$$

$X(t)$ and $Y(t)$ are defined as

$$X(t) = \Delta\cos[2\mu E_0 t + (2\mu E/\omega)\sin(\omega t)] \tag{10}$$
$$Y(t) = -\Delta\sin[2\mu E_0 t + (2\mu E/\omega)\sin(\omega t)].$$

Note that from equation (4) we have always $|\Theta(T)| \leqslant \pi$. This means that the quasienergy should be in the range $-\omega/2 \leqslant \epsilon \leqslant \omega/2$, or equivalently the quasienergy gap $\Delta\epsilon = \epsilon_+ - \epsilon_- \leqslant \omega$. So, the length of a Brillouin-zone in the quasienergy space is ω. We call the range $-\omega/2 \leqslant \epsilon \leqslant \omega/2$ the first Brillouin zone. Other Brillouin zones can be obtained by adding integral multiples of ω to the quasienergy. With this notation, and by straightforward calculations from equations (3)–(10), we find that in the case of high-frequency driving field ($\Delta/\omega \ll 1$), the exact crossing of quasienergies occurs when

$$\frac{2\mu E_0}{\omega} = n \qquad (n = 0, 1, 2, \ldots) \tag{11}$$

and

$$\frac{2\mu E}{\omega} = x_n \tag{12}$$

simultaneously, where x_n is a non-zero root of the ordinary Bessel function of nth order J_n, i.e.,

$$J_n(x_n) = 0. \tag{13}$$

Needless to say, the $n = 0$ in equation (13) is nothing but the reflection of the coherent destruction of tunnelling for the pure AC-driven two-level dynamics discussed by Grossmann and Hänggi [2]. There the one-dimensional manifold in the parameter space (ω, E) for the appearance of the accidental degeneracy of two quasienergies was found to be described by equation (13) in the special form $J_0(x_0) = 0$. Therefore, in the following discussion, we will ignore this situation and focus on the biased case ($n \neq 0$), i.e., the situation of the AC–DC driving field.

Note that from equations (11)–(13), there is essentially one constraint condition in the parameter space (E, E_0, ω) to create an accidental degeneracy of quasienergies. Therefore, one can vary two parameters independently to fulfil this condition. This observation is coincident with the argument of von Neumann and Vigner [11] because the degeneracies investigated here have codimension two, as generally discussed by Berry [12]. Figure 1 shows the exact crossing of quasienergies in the section (E, E_0) through the parameter space (E, E_0, ω), where the fan lines describe the constraint between E_0 and E to observe the exact crossing of quasienergies. Note that only the case of $n = 1$–10 has been drawn in this figure, and the corresponding values of x_n have been chosen as the first non-zero roots of the ordinary Bessel functions J_n, i.e., $x_1 = 3.832$, $x_2 = 5.136$, $x_3 = 6.380$, $x_4 = 7.588$, $x_5 = 8.772$, $x_6 = 9.936$, $x_7 = 11.086$, $x_8 = 12.225$, $x_9 = 13.350$, and $x_{10} = 14.482$. Obviously, the exact quasienergy crossing should occur as long as the DC field strength E_0 and the AC field strength E locate at the fan.

Also, we should notice that, as has already been indicated in [10], the phenomenon of exact quasienergy crossing implies that the coherent tunnelling of the electron in the undriven system may completely turn into that of localized motion through the choice of the field parameters when the driving field is switched on. Therefore, the level crossing condition (13) represents at least one of the dynamic localizations of coherent motion. This observation is confirmed by our numerics depicted in figure 2(a), where the evolution probability $P(\tau)(= |\psi^\dagger(0)\psi(\tau)|^2)$ for the electron in the initially occupied left well, i.e., $\psi^\dagger(0) = (1/\sqrt{2},\ -1/\sqrt{2})$, is plotted as a function of the reduced driving time τ ($= \omega t$). In our calculation, we have taken $\Delta/\omega = 0.1$. Note that only the case of $n = 1, 2, 3$, and correspondingly, $x_1 = 3.832$, $x_2 = 5.136$, and $x_3 = 6.380$ has been exhibited in this figure. There the solid line shows the $P(\tau)$ for $n = 1$ and $x_1 = 3.832$, while the dashed and the dot–dash lines show the $P(\tau)$ for $n = 2$, $x_2 = 5.136$, and for $n = 3$, $x_3 = 6.380$, respectively:

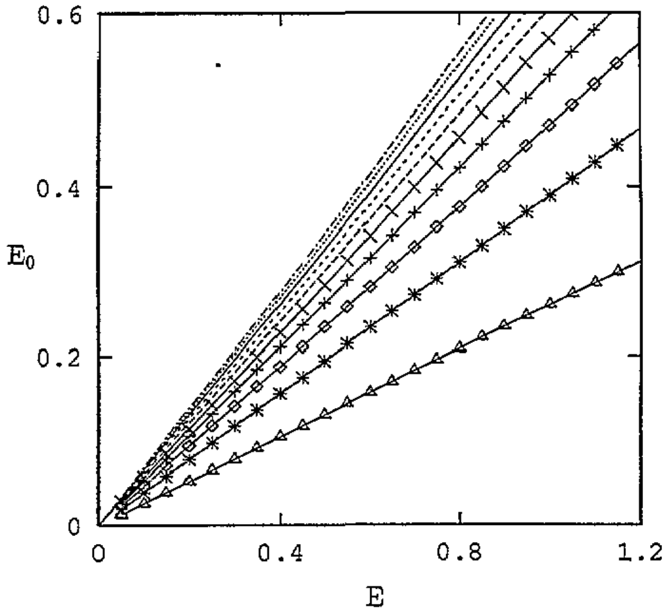

Figure 1. The fan structure representing the exact crossing of quasienergies in the section $(E,\ E_0)$ through the parameter space $(E,\ E_0,\ \omega)$: Δ, $n = 1$, $x_1 = 3.832$; $*$, $n = 2$, $x_2 = 5.136$; $\Diamond$, $n = 3$, $x_3 = 6.380$; $+$, $n = 4$, $x_4 = 7.588$; $\times$, $n = 5$, $x_5 = 8.772$; – – –, $n = 6$, $x_6 = 9.936$; - - - -, $n = 7$, $x_7 = 11.086$; ——, $n = 8$, $x_8 = 12.225$; · · · · · ·, $n = 9$, $x_9 = 13.350$; — · —, $n = 10$, $x_{10} = 14.482$.

they do in fact touch. From figure 2(a), one can clearly see that $P(\tau) \simeq 1$ (> 0.97) for any period of the driving laser, which gives the impression that the electron is frozen in the initially occupied left well. However, if equation (13) cannot be met, the situation will be different. In that case, the frozen state will be melted, and the electron will visit either well through the quantum tunnelling. This feature has been manifested in figure 2(b), where the curves exhibit the coherent-tunnelling probability for the electron in the initially occupied left well. Here the solid line denotes the $P(\tau)$ for $n = 1$ and $x_1 = 2.5$, while the dashed and the dot–dash lines denote the $P(\tau)$ for $n = 2$, $x_2 = 3.6$ and for $n = 3$, $x_3 = 4.4$, respectively. Obviously, the $P(\tau)$ oscillates between zero and unity, implying that the electron tunnels between the two wells. That $P(\tau) = 1$ means that the electron is only located in the left well. On the contrary, when $P(\tau) = 0$, the electron will be fully located in the right well.

Another feature we noticed in figure 2(a) is that although the electron is almost completely located in the initially occupied left well, there is still a small but obvious deviation from unity. The reason is that when deriving the level-crossing condition (13) we have assumed the frequency of the driving laser field is high enough so that $\Delta \ll \omega$, but when plotting figure 2(a), we have only taken $\Delta = 0.1\omega$. Hence, it is expected that such deviations will gradually disappear if we choose a smaller ratio Δ/ω. This can be clearly seen from figure 3, where we take $\Delta = 0.05\omega$ in figure 3(a) and $\Delta = 0.01\omega$ in figure 3(b) respectively.

In summary we have investigated the quantum behaviour of electrons in a two-level system under the influence of an AC–DC field. By means of the technique of $SU(2)$ Lie algebra, we have obtained the closed-form solution for the quasienergy, from which the exact

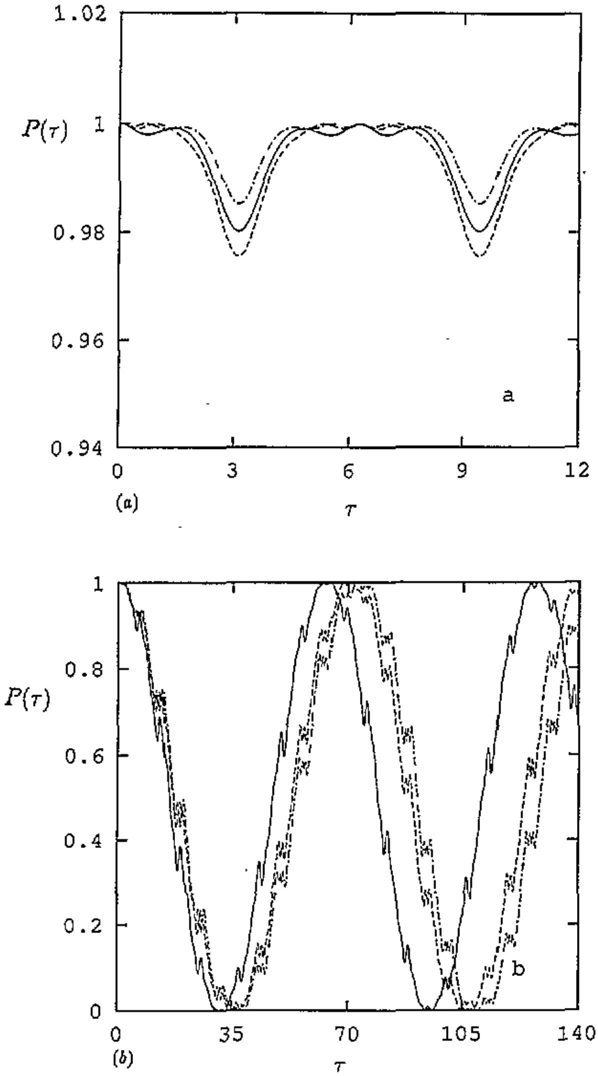

Figure 2. The evolution probability $P(\tau)$ for the electron in the initially occupied left well plotted as a function of the scaled driving time τ $(= \omega t)$. In (a), the solid line denotes the case of $n = 1$ and $x_1 = 3.832$, while the dashed and the dot–dash lines denote, respectively, the cases of $n = 2$, $x_2 = 5.136$ and $n = 3$, $x_3 = 6.380$. In (b), the solid line corresponds to $n = 1$, $x_1 = 2.5$, while the dashed and the dot–dash lines correspond to $n = 2$, $x_2 = 3.6$ and $n = 3$, $x_3 = 4.4$ respectively.

level crossing has been found to appear in the circumstance of the constraint condition (13) being satisfied if the high-frequency driving field is involved. It has also been demonstrated that the occurrence of the exact crossing of quasienergies coincides with the onset of coherent destruction of tunnelling. Such an effect of dynamic localization should be observable in experiments by using free-electron laser facilities.

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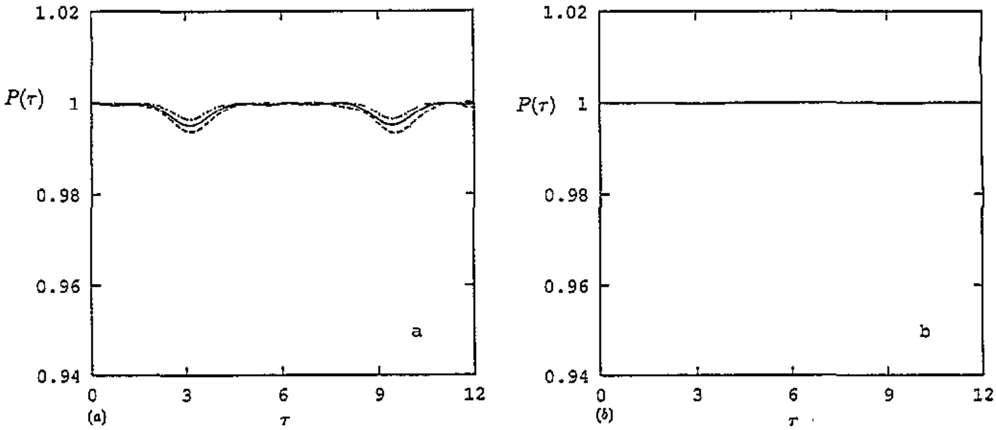

Figure 3. The evolution probability $P(\tau)$ for the electron in the initially occupied left well plotted as a function of the scaled driving time τ $(=\omega t)$. The ratio Δ/ω is chosen as 0.05 in (a) and 0.01 in (b) for each line. In both (a) and (b), the solid line denotes the case of $n=1$ and $x_1 = 3.832$, while the dashed and the dot-and-dash lines denote, respectively, the cases of $n=2$, $x_2 = 5.136$, and $n=3$, $x_3 = 6.380$.

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